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Understanding the Probability of Risk Management through Quantitative Modelling In the Commercial Business

Abstract

This research presents the risk management optimization model, through the identification of a hedge strategy which maximizes the hope of profits. The model is based on the premise that risk factors, besides affecting the results of companies, may, also be correlated with the opportunities investment. The proposed model incorporates the costs of indebtedness as well as the benefits of availability resources. The work is developed as follows: (1) discussion of the aspects related to the topic of risk management in the context of finance theory; (2) presentation the methodology adopted for the development of of model; (3) derivation of the fundamental steps that guided the mathematical model, highlighting the specificities of the relevant variables and simulation mechanisms and (4) presentation of main results of the optimal hedge model, among which the following stand out: increased hope of profits, reducing uncertainty in relation to investments, greater stability of the optimal level of investment and indebtedness and lower of the company's results, due to the of the level of financial risk.

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Introduction

The issue of risk management is becoming increasingly important in the business context. With the increasing interdependence of markets, companies become more vulnerable to various risk factors. Economic, financial and even competitive movements are rapidly spreading, which can considerably affect company results. Assuming that the capital market is perfect and information is symmetrical, the theorems formulated by Pratt (2010) can be adapted to demonstrate that risk management is not relevant to the value of the firm. In this situation, if investment strategies are defined and independent of financing decisions, the creation of shareholder value results from the adequate use of investment opportunities in real projects. But when you look at the real world, some premises of Pratt are not obeyed, and so the practice of risk management can effectively generate value. Potential agency conflicts arising from information asymmetry problems are market imperfections that justify the use of risk management mechanisms. Knowing that access to information is not uniform and assuming that agents are willing to maximize their own usefulness to the detriment of the wealth of principals, risk management becomes relevant by signalling to the market smaller agency problems.

Unexpected fluctuations in risk factors can lead to levels of indebtedness and liquidity that influence the value of the company. In situations of high use of third-party capital, the various stakeholders can penalize the company by requiring greater disclosure of information, establishing monitoring and control mechanisms and imposing costs arising from the probability of bankruptcy due to the company's level of indebtedness. On the other hand, stakeholders can pick up positive signals and cause an increase in value for the company. Greater liquidity, for example, can facilitate access by the company to sources of funding if domestic funds are committed to investment projects. Thus, risk management programs that avoid excessive debt or protect corporate liquidity can be sources of wealth generation. This research is based on the risk management in which indebtedness represents costly capital for the company. The model is based on the premise that risk management is beneficial as unnecessary fluctuations in investment and indebtedness can be avoided. These fluctuations are due to a single risk factor that may, for

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example, correspond to the return of the product traded by the company or to the return of an index of assets or liabilities. Advancing the theoretical knowledge on risk management, the benefits associated with liquidity will also be considered in this research.

The proposed model therefore incorporates the liquidity issue through the assumption that the level of net assets, represented by the availability of internal resources, can bring value benefits. This innovation of the model allows for the joint analysis of virtually all relevant financial issues. The model makes it possible to directly evaluate investment decisions, indebtedness, liquidity and risk management and, indirectly, the decision on dividends. Thus, in the model presented, the investment decision is taken into account by identifying that the investment opportunities in projects are dependent on the achievements of the risk factor. The issue of financing is associated with the need for third-party capital which, coupled with internally generated resources, complements the amount of investments that maximizes the company's profit. In the model, external financing presents costs that decrease the creation of value provided by the new projects. The model also evaluates the benefits arising from the existence of internal resources that can eventually be used for investments. In this case, the liquidity of the company is a source of wealth creation. Indirectly, the question of dividends can also be considered in the model, since internally generated resources would represent flows available for shareholder remuneration. Finally, the issue of risk management is incorporated in the study, since the modelling allows the identification of an optimal hedge level of internal resources that maximizes the expected value of the company's profit.

Theoretical References

The theory of risk management is based on practically two approaches. The traditional approach to hedge in non-financial corporations, which analyses the use of futures contracts for the protection of financial exposures. Subsequently, Jovanovic (2014) popularized the approach by investigating investments in futures contracts as a way of counterbalancing risk. The traditional approach assumes that the company, by behaving as a risk-averse investor, seeks to minimize risk. The optimal number of futures contracts to be transacted is estimated based on the risk minimization of the hedged portfolio. According to simplifying assumptions, the hedge optimal is given by the covariance quotient between the fluctuations of the spot and future prices with the variance of the future price fluctuation. The more modern approach to risk management takes into account the studies of Martin (2008) on the contractual relations existing in a company. Due to possible conflicts of interest between managers, shareholders, creditors and other people associated with the company, the need for risk management may depend on the relationships between the stake holders, arising from the form of remuneration, information asymmetry, risk profile, etc. Modern risk management optimization models therefore involve the identification of assumptions that regulate agency relationships or conflicts of interest between the characters involved in the management and financing of a corporation.

Traditional models can be considered as particular cases of modern models, since several optimization models, which involve the identification of optimal hedge indices, are indirectly based on agency relationships. According to Bouchaud (2011), if the company has behaviour similar to a trader which shows risk aversion and seeks to maximize its usefulness, then there is an optimal value of the relationship between the exposure in a futures contract and the exposure in a cash position that minimizes the risk of the total portfolio. The company's risk minimization may be associated with agency problems, as it indirectly reduces the personal exposure of managers because of the greater stability of cash flows. Thus, considering that the probability of financial problems arising from hedging is considerably low, administrators can enjoy greater stability in their positions, such as by maintaining their jobs. For the company manager, this risk reduction is rational, as it generally does not have a very diversified investment portfolio Reilly (2012) and, therefore, the value obtained through its wages may represent a considerable portion of its wealth. Fong (2005) obtain optimal hedge indices from two different sets of assumptions. Assuming that the returns to spot prices and futures prices have a bivariate normal joint distribution, the authors derive the hedge indices from least-squares regression and predetermined utility functions. Additionally, if there is the possibility of raising and applying at a risk-free interest rate, then the optimal hedge index is independent of the investors' preferences and can be determined by the parameters of the joint distribution. On the other hand, studies the optimal index of hedge from a non-tradable position in financial markets through an intertemporal analysis, assuming that spot and futures prices change continuously and that investors can adjust their consumption and hedging behaviour at all times.

Bos (2007) compares two approaches for estimating optimal hedge indices that vary over time. Through the study of variances and covariance of agricultural commodities, the author establishes the conditioning of information with different degrees of sophistication. Thus, in the first approach, mobile sample variances and covariance's are calculated from the prediction errors of spot prices and future prices. In the second approach, estimates of variances and covariance are obtained through generalized autoregressive models of conditional heteroscedasticity. Strong evidence is found that the optimal hedge indexes vary over time, but the results do not show better performance of sophisticated models in the estimation of optimal hedge indices. Epstein (2012) presents the maximization of company value through the evaluation of the trade-off between the reduction in the expected costs of bankruptcy and the increase in the expected costs of the exchange rate hedge. From a hedge- based approach that minimizes the probability of bankruptcy within a predetermined time interval, the authors evaluate an index between cost and benefit of the hedge, where the benefit is defined as the expected reduction in the cost of bankruptcy provided by the hedge and the cost is defined as a direct cost of establishing the policy hedge. Dixon (2012) analyses policies of hedge under the premise of maximizing the expected utility of management, taking into account the impacts of exchange rate fluctuations and hedge costs on risk management policies.

Kuruc (2003) identify hedging strategies depending on the way in which the expected utility of the administrator is linked to the value of the company and the way in which the managers are rewarded. To avoid the use of free cash flows for non-pecuniary benefits by management, the company could exchange equity for third-party capital by creating mechanisms that discipline managers in relation to their future investment decisions. Since higher indebtedness can lead to costs of financial hardship, risk management enables a greater degree of certainty of project cash flows and, consequently, reduction of bankruptcy costs would serve as prevention against non-pecuniary expenses Simonovic (2012). Berrospide (2008) identifies an optimal hedge portfolio that allows the maximization of the company's value, in which risk management enables an increase in the optimal indebtedness index, allowing the company to obtain greater tax benefits. In this case, risk management has an indirect or secondary role, since hedge brings second-order increases in shareholder wealth. In this case, the increase in shareholder value comes mainly from the greater possibility of leverage. Finally, establish an analytical framework in which risk management represents an interface between investment and financing policies. In this way, the hedge optimal is a function of the investment opportunities available and the company's ability to obtain external financing. The study allows not only the explanation of the reasons for a company to carry out the hedge, but also the identification of the amount and type of hedge to be implemented.

Empirical Studies

Empirical studies on risk management in nonfinancial corporations are mainly associated with research on the use of derivatives, since they are typical instruments for reducing or increasing exposure to risks. The empirical analyses related to the use of derivatives depend strongly on a broad data collection, through which, in most cases, subjective evaluations of the researchers are necessary, mainly for the characterization of categorical data. In particular, the identification of a user or non-user company of derivatives may vary between different surveys, due to the data collection form or the preliminary definitions of the researchers. Chance (2013) classify companies, in relation to the use of derivatives, by searching financial databases for certain keywords, such as hedging, swaps or options. Beil (2013) classify derivative users by searching for references on the use of exchange rate derivatives in the financial statements of companies belonging to the S & P 500. In turn, examines whether the company uses derivatives to manage risks. Thus, companies are classified based on the analysis of information regarding the use of derivatives, as well as the identification of accounting treatment compatible with hedge transactions.

Given the characteristics of the data collection process, the empirical tests of risk management theory basically use two different approaches. The first approach measures the effect of the use of derivatives in the financial exposures of companies. The second approach is based on the comparative analysis of the characteristics of companies that use and do not use derivatives. With respect to the comparative studies, Bodnar (1998) observe that the use of derivatives is positively associated with the total turnover of the company. On the other hand, more than half of the survey respondents, conducted with US non-financial corporations, claim not to use derivatives because the level of risk is considered low or can be managed in other ways. Among the companies that use derivatives, the main objectives are to reduce risks and reduce financing costs. Few companies use derivatives for speculative purposes, clearly demonstrating management's willingness to use the derivatives market to protect its exposures to variables that are difficult to control Bodnar (1998). It identifies that larger Australian firms are more likely to use derivatives based on transaction costs. In addition, the higher the indebtedness and the greater the dividend payment, the more hedge companies perform.

In comparative studies, it's note that New Zealand firms have even more active derivative management than US companies, and also provide reporting mechanisms for the use of more informative derivatives. One reason for this evidence lies in the fact that New Zealand firms are more exposed to currency risk and therefore more concerned about managing that risk. Bodnar (1998) also obtain evidence that companies in some situations hedge derivatives, but imperfectly. In some companies analysed, speculative positions seem to be taken as if they were hedge transactions. Studies for the mining industry have made possible more specific analyses that go beyond the categorical data evaluation. Dionne (2000) conducts a detailed study on risk management in gold mining companies. The main result of the study is based on the hypothesis that companies use derivatives to reduce risks. There is evidence that the primary motivation for the establishment of hedge operations involves the risk aversion of shareholders and managers. In the study of Dionne (2000), the determinants of the extent of the hedge executed by the company are examined. The author discovers a strongly negative relationship between the extent of hedge use and the liquidity of gold mining companies.

Canabarro (2009) identify that the choice of the risk management method, whether through the use of derivatives on gold or through of other operations, such as gold-indexed loans, depends on differences in the ability of firms to adjust operating costs and different capital requirements for investments. In addition, the authors also identify that management's incentives to use derivatives to reduce risk depend on how executives are rewarded. When studying oil and gas producers' evidence, Jin (2006) identifies the relationship between hedge and funding costs. In this way, the author observes that the probability of hedging is associated with the economies of scale of hedging costs and the basic risk of hedging instruments. Companies more prone to risk management are larger and have production coming mainly from regions where the product extracted has a high correlation with assets objects to which derivatives traded on the Stock Exchange are linked. Kenyon (2012) report that firms are less likely to use forward and futures contracts to manage exchange rate risk when their liquidity is high and when financial difficulties are not a potential threat. In addition, the probability of using derivatives is low when the R & D expenditure ratio is low, corroborating results by Nersesian (2011).

Research Methodology

In the theoretical reference, the approaches on risk management were presented and discussed, aiming to substantiate the importance of the proposed model. From the identification of variables relevant for the valuation of companies, a quantitative model was developed, in which the aim was to evaluate the level of hedge that maximizes the expected profits of the company. Through the integrated analysis of risk management, investment opportunities, borrowing needs and the availability of internal resources. The introduction of the liquidity variable, a relevant financial aspect for risk management, makes the model more complete and also brings greater complexity in mathematical derivations. From the formulation of the hedge level optimal, the possible results of "hedged" and non-hedged companies were simulated to show the gains of risk management strategies.

Model Development

As discussed previously, according to certain premises analogous to those of, indebtedness decision, dividend policy, liquidity management and risk management do little to contribute to shareholder value creation. Under these conditions, only the investment decision would be relevant. However, when breaches of the assumptions are found, more sophisticated financial models, which relax various assumptions, can provide subsidies for financial practice, justifying the importance of the various aspects of financial management. In the context discussed in this paper, the profit function P can be defined as the net present value F of the investment opportunities, subtracted from the costs C from the indebtedness necessary for the implementation of the projects and added by the benefit B of the liquidity arising from the existence of internal resources which can finance part of the investments, that is,

$$P(w) = F(I) - C(e) + B(w), \text{ em que } I = e + w \qquad (1)$$

being that:

I = represents the level of investment;

e = is associated with additional funding;

w = corresponds to internally generated flows.

Thus, the investment in projects is financed by internal resources (w) and indebtedness with third-party capital (e). Assuming that the risk factor to which the company is exposed influences the investment opportunities, then the net present value can be established by the following equation:

$$F(I) = \theta f(I) - I, \text{ com } \theta = p(\varepsilon - I) + I. \qquad (2)$$

being that:

ρ = a measure of the correlation between investment opportunities and the risk factor and

ε = the return of the risk factor that influences the company's result; for purposes of mathematical simplicity, it will be considered that the risk factor has a normal distribution with mean 1 and variance σ 2.

In the proposed model, f (I) will be defined as the function that associates, for each investment amount I, a present value of the new projects. Given that viable investment opportunities are depleted by specific constraints of firms, such as technology, competition, it will be considered that:

$$\frac{df(I)}{dI} = f_1 > 0 \text{ e } \frac{df_1}{dI} = f_{11} < 0 \qquad (3)$$

That is, the value creation of investments is increasing, but additional investments lead to ever smaller increases in the present value of the projects.

It is important to observe, as discussed previously, that the net present value of the projects is influenced by the risk factor ε. If the correlation ρ is positive, then situations in which the risk factor exceeds its expected value increase the present values of the investment opportunities. When the realized risk factor is below its average value, investment opportunities tend to lose value. The case where ρ is greater than zero can be exemplified by an exporting company, subject to the risk of exchange rate fluctuations. Thus, an increase in the dollar could positively influence business possibilities, making potential projects lead to higher present values. In the situation where the correlation ρ is negative, if the realized value of the risk factor is lower than its average, indicating a result below the expected, then the investment opportunities involve higher present values. In the opposite situation, if the risk factor is higher than the expected value, the investment opportunities lead to lower present values. An example for the negative correlation case can be represented by a company exposed to exchange rate risk, where a fall in the exchange rate reduces internally generated cash flows, but at the same time implies better investment opportunities for the future.

Debt costs can come from several sources. These costs can arise from loss of value associated with an increase in financial difficulties and an increase in the potential for bankruptcy of the company. Thus, higher indebtedness can lead to direct costs in the form of payment for legal services in the event of bankruptcy and indirect costs in the form of a decrease in the company's competitiveness and operational inefficiency that may arise, for example, with the issue of underinvestment. In addition, the asymmetry of information between managers and external investors may also imply indebtedness costs. Companies with adequate investment opportunities, but with a level of indebtedness considered unsatisfactory, may have their value diminished by the market, due to the informational difference. Agency problems, too, can lead to debt costs. For example, shareholders incur disbursements to monitor the behaviour of managers regarding the level of debt. In addition, managers can obtain private gains by limiting third-party capital by working on non-optimized debt levels.

Considering that excessive indebtedness can considerably increase bankruptcy costs, then the function C (e) is modelled as follows:

$$\frac{dC(e)}{de} = C_e > 0 \text{ e } \frac{dC_e}{de} = C_{ee} > 0 \quad (4)$$

As a result, borrowing costs are increasing and increases in indebtedness cause increasing costs.

The benefit of liquidity is based on the attempt to reduce information asymmetry. Signalling through the use of own resources to make investments gives greater security to external investors. In this way, the use of internal resources signals the commitment of capital to the projects, reducing the penalty imposed by the market due to the need for greater indebtedness. Agency disputes between shareholders and creditors are also ameliorated by a greater liquidity of the company, since the directing of internal resources for investments may suggest that the implementation of projects generates more wealth than the distribution of dividends to shareholders.

Assuming that above a certain limit liquidity brings negligible marginal benefits to the firm, then reasonable conditions for B (w) can be described by:

$$\frac{dB(w)}{dw} = B_w > 0 \text{ e } \frac{dB_w}{dw} = B_{ww} > 0 \quad (5)$$

It is important to note that the modelling excludes potential penalties imposed on the company for excess liquidity, which could signal a lack of investment opportunities or excessive levels of conservatism on the part of the managers.

Having defined the relevant functions, the next step is to establish the financial dynamics. The proposed model consists of the analysis of three instants: t_0, t_1 et$_2$. Starting the evaluation from the last period, one can identify the relevance of the risk management strategy to be defined in the initial period. Thus, at t_2, the firm obtains a result that is a function of the optimal investment level at t_1. The investment established in t_1, together with the availability of the company's internal resources in that period, defines the additional indebtedness required.

Whereas risk factor achievements can change the net assets of the company to be used for investments, due in t 0identify what fraction of the initial internal availability must be protected. Therefore, the hedge decision occurs at t_0, implying that the optimization mechanism is based on the expected value or the expected profit function, due to the uncertainty associated with the risk factor at the initial time.

Following the dynamics described above, in t_1 the company must identify the level of investment that maximizes profit. Thus, the optimization problem is given by:

$$\max P(w) \quad (6)$$

Deriving the profit function in relation to investment, we have:

$$\frac{dP}{dI} = \frac{d(\theta f(I) - I)}{dI} - \frac{\partial C(e)}{\partial e}\frac{\partial e}{\partial I} + \frac{\partial B}{\partial W}\frac{\partial W}{\partial I} \quad (7)$$

Since at time t_1 the risk factor has already been done and therefore can be observed, then we θ are determined, i.e., w being fixed, changes in investment are fully funded by variations in the level of debt. Therefore, the first and second order conditions establish that the optimal level of investment is given by:

$$\frac{dP}{dI} = 0 \Rightarrow \theta f_1 - I - Ce = 0e$$

$$(8)$$

$$\frac{d^2P}{dI^2} < 0 \quad \theta f_{11} - C_{ee} = 0$$

As discussed previously, the hedge issue arises at t_0, since the value of the internal resources that will be available at t_1 is associated with the risk factor ε. In the model, at t_0 the company has resources w_0 that are subject to value fluctuations depending on the risk factor. The company can choose t_0 conduct a hedge w_0 in such a way to eliminate some of the uncertainty of domestic remedies to be used in wt_1 to finance great investment. Thus, the risk management decision will involve the establishment of the percentage h of domestic remedies w_0 which will be protected against fluctuations of the risk factor ε.

Considering the risk factor ε, the internal resources w at t_1 can be modelled by:

$$w = w_0(h + (I - h)\,\varepsilon) \quad (9)$$

on what:

$w_0 \neq 0$ = represents the amount of internal resources available at t_0 and

h = represents the hedge index.

It is easy to identify two special cases. If h = 1, that is, if the hedge is complete, $w = w_0$, and thus there is no uncertainty about the internal resources available at t_1. If h = 0, then $w = w_0\varepsilon$, and therefore the internal resources at t_1 will reflect all possible variability of the risk factor ε. Depending on the form and parameters of the present value function of the investments, the cost function associated with the indebtedness and the function of liquidity benefits, the optimal hedge level may be different from these special cases. Moreover, in a preliminary analysis, considering the possible values of the correlation ρ between the risk factor and the investment opportunities, one cannot rule out the possibility that h has values outside the range [0, 1]. Values larger than the unit reflect that the hedge is made on an amount higher than the initially available internal resources. Negative values imply that risk management leaves domestic resources more vulnerable to fluctuations in the risk factor. In this case, risk management suggests an increase in risk exposure, so that investment opportunities are better utilized.

Considering that in t_0ε is still not defined, in order to obtain the optimal hedge formula, the optimization problem must be solved, identifying h * such that the expectation of the profit function is maximal. Thus, the optimization problem is given by:

$$\max_{h} E_0[P(w(\varepsilon,h)] \qquad (10)$$

Thus, as a function of the randomness of the risk factor, the expected value of the profit function, measured at time t_0, will be maximized. Considering the assumptions of the model, maximization involves the following first-order condition:

$$\frac{dE_0[P(w(\varepsilon, h))]}{dh} = 0 \qquad (11)$$

If x and y are random variables with bivariate normal distribution e.g. (y) a differentiable function in relation to y, with $|g(y)| \leq k < \infty$, then Cov [x, g (y)] = E [g '(y)] Cov (x, y). Using the theorem, where x and y are equal to ε, and considering that the derivative of profit in relation to internal resources is also differentiable with respect to ε, it can be shown that the first-order condition can be described by:

$$E_0\left[\frac{d\left(\frac{\partial P}{\partial w}\right)}{d\varepsilon}\right] = 0 \qquad (12)$$

Solving the above equation, it is shown that:

$$\frac{d\left(\frac{\partial P}{\partial w}\right)}{d\varepsilon} = pf_I \frac{\partial I}{\partial w} + \theta \frac{\partial f_I}{\partial I} \frac{\partial w}{\partial \varepsilon}\left(\frac{\partial I}{\partial w}\right)^2 + (\theta f_I - 1)\frac{\partial^2 I}{\partial w^2}\frac{\partial w}{\partial \varepsilon} - C_{ee}\frac{\partial w}{\partial \varepsilon}\left(\frac{\partial e}{\partial w}\right)^2 - C_e \frac{\partial^2 e}{\partial w^2}\frac{\partial w}{\partial \varepsilon} + B_{ww}\frac{\partial w}{\partial \varepsilon} \qquad (13)$$

Finally, using the implicit function theorem, through mathematical manipulations, one obtains the result of the level of hedge that maximizes the expected profit of the company.

$$h = I + \frac{p}{w_0}\frac{E_0\left[\frac{f_I C_{ee}}{\theta f_{II} - C_{ee}}\right]}{E_0\left[\frac{\theta f_{II} C_{ee}}{\theta f_{II} - C_{ee}} + B_{ww}\right]} \qquad (14)$$

Simulation of the Model

The analytical formula given by equation 14 makes it possible to identify the optimal hedge level to maximize expected profit. In the model, investment opportunities vary depending on a risk factor that also affects the availability of internal resources. The risk management strategy therefore involves the definition of an asset hedge index that maximizes the company's expected profit, taking into account the present value of the investments, the borrowing costs and the liquidity benefits.

Since the optimal hedge formula is generic and complex, it becomes difficult to assess how the various characteristics associated with the company can influence the risk management decision. The simulation performed in this section allows, through the establishment of specific functions, the evaluation of the sensitivity of the hedge strategy to changes in the different factors relevant to risk management. The behaviour of the optimal risk management mechanisms will be investigated considering Cobb-Douglas type functions to calculate the variables on which the expected profit function depends. These specific functions were chosen for the ease of mathematical treatment and the flexibility in the construction of models that present useful and realistic behaviour. Also, uses Cobb-Douglas functions obtaining, through the Taylor expansion of second order, an approximate analytic formula for the hedge optimal, in the event that there is no liquidity benefit. In this research, instead of looking for results of analytically approximated expected value, several achievements of the risk factor will be simulated, so that the hopes of the equation 14 can be estimated through the sample means of the random variables.

Equation 14 represents the hedge index that maximizes the expectation of the profit function. The functions f (I), C (e) and B (w) must be such that the first and second order conditions described by equation 8 can be verified. The result of the optimal hedge must obey the system defined by equations 1, 8 and 14.

For the simulations, initially, the function that represents the present values of the investment projects will be defined:

$$f(I) = \frac{\phi_I I^{I-\beta_I}}{I - \beta_I} \qquad (15)$$

where φ_1 and β_1 are constants, such that $\varphi 1 > 0$ is a factor that adjusts the present value function to the scale of business opportunities of the company; $0 < \beta_1 < 1$ is a parameter associated to the elasticity of the present value of business opportunities in relation to the investment decision.

In fact, $1 - \beta_1$ represents a concept of elasticity, since:

$$\frac{\frac{df(I)}{f(I)}}{\frac{dI}{I}} = \frac{df(I)}{dI}\frac{I}{f(I)} = \phi_I I^{-\beta_1}\frac{I}{\frac{\phi_1 I^{I} - \beta_I}{I-\beta}} = I - \beta_I \qquad (16)$$

In addition, the cost function can be defined as:

$$C(e) = \frac{\phi_2 e^{I+\beta_2}}{I + \beta_2} \qquad (17)$$

where φ_2 and β_2 are constants, such that $\varphi_2 > 0$ represents a scale factor that corrects the costs of third-party debt to company characteristics; $0 < \beta_2 < 1$ is associated with the elasticity of cost in relation to the amount of money borrowed.

Finally, the benefits arising from the company's liquidity are defined as:

$$B(w) = \frac{\phi_3 w^{I-\beta_3}}{I + \beta_3} \qquad (18)$$

where φ_3 and β_3 are such that $\varphi_3 > 0$ is a scale factor that makes it possible to adjust the level of liquidity benefit for different types of companies or for differentiated market conditions; $0 < \beta_3 < 1$ is a parameter representative of the elasticity of the liquidity benefit in relation to the availability of internal resources.

This specification of the functions f (I), C (e) and B (w) allows the definition of the system of equations that must be observed. In the case of the non- hedged company, the optimal investment level must satisfy the first-order condition of equation 8 and the equilibrium relation between investments and sources of financing presented in equation 1. Considering the conditions imposed in the simulation model, there must be, simultaneously, for the non-hedged company:

$$\theta\phi_1 I^{-\beta_1} - I - \phi_2 e^{\beta_2} = 0 \text{ e } I = e + w_0\varepsilon \qquad (19)$$

In turn, the "hedged" company must, in addition to satisfying the first-order condition given in equation 19, obey the equilibrium equation of investments and sources of resources and the optimal hedge equation. The equilibrium equation is given by:

$$I = e + w_0(h + (I - h)\varepsilon) \qquad (20)$$

The optimal hedge equation for the Cobb-Douglas functions can be obtained by replacing the results of the derivatives of the functions f (I), C (e) and B (w) in equation 14:

$$h = I + \frac{p}{w_0}\frac{E_0\left[\frac{\phi_1\phi 2\beta_2 I^{-\beta_1}e^{\beta_2-1}}{\theta\phi_1\beta_1 I^{-I-\beta_1} + \phi_2\beta_2 e^{\beta_2-1}}\right]}{E_0\left[\frac{-\theta\phi_1\phi_2\beta_1\beta_2 I^{-\beta_1-1}e^{\beta_2-1}}{\theta\phi_1\beta_1^{-1-\beta_2} + \phi_2\beta_2 e^{\beta_2-1}} - \phi_3\beta_3 w - {}^{\beta_3-1}\right]} \qquad (21)$$

Thus, in practical terms, the estimation of the optimal hedge is obtained through a recursive procedure based on a computational algorithm that follows the following steps: (1) generation of the risk factor realizations, (2) definition of the parameters of the functions and (3) calculating the optimal level of investments according to equation 8, (4) calculating the level of indebtedness and the level of internal resources, (5) calculating auxiliary parameters representative of the expectations of equation 21, (6) iteration of the hedge level given by equation 21, (7) checking the stopping criterion that defines the need for additional iterations to refine the hedge estimate. In the procedure, risk factor achievements are simulated and the hopes are obtained through the sample means of the realizations. In addition, numerical calculation mechanisms are used to identify roots of nonlinear equations.

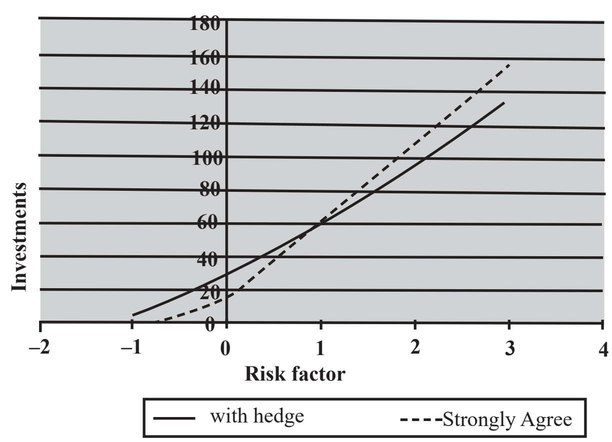
Evaluation of Results

For the simulation, parameters of the present value, cost of indebtedness and liquidity benefit functions were identified that led to values of similar order of magnitude and that allowed for relevant comparative analyses. In addition, in order to better adapt to reality, the parameters imply that investment opportunities are more important than the costs and benefits considered in modelling. The Table 1 shows the parameters used in the simulation initially performed to investigate the impact of hedging the company's results, considering different risk factor achievements. In this simulation, it was established that the correlation between the risk factor and the investment opportunities is positive.

Table 1. Parameters of the simulation model for analysis of profit levels and investment needs

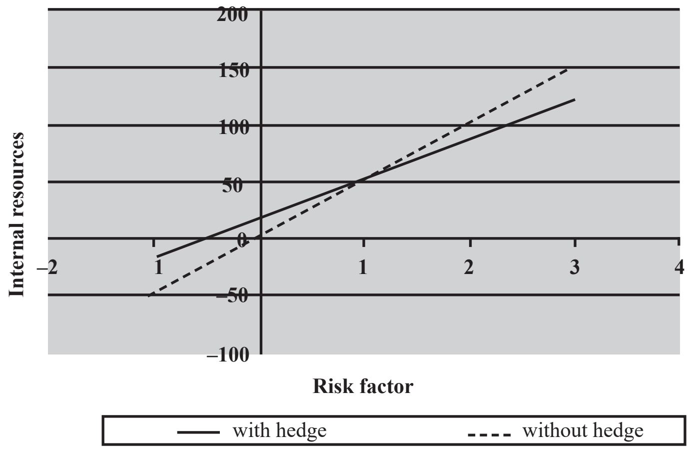
Variable	Parameter	Value
Risk Factor (ε)	μ	1
	σ	0,2
	ρ	0,1
Present value : $f(I)$	β_1	0,25
	ϕ_1	20
Cost of debt : $C(e)$	β_2	0,5
	ϕ_2	2
Benefits of liquidity : $B(w)$	β_3	0,1
	ϕ_3	0,2
Internal resources (W)	w_0	50
Initial value	I_0	20
	h_0	0,5

Under the conditions defined in the previous table, the results, described in Graph 1, of the optimal investment volume were obtained according to the different achievements of the risk factor. Two cases were studied: in the first case, it was considered that the company adopts an optimized hedge strategy and, in the second, it was considered that the company does not establish any risk management program. It is important to note that the hedge soothes the behaviour of the optimal investment level. The lower range of variation for the case of the "hedged" company shows that risk management implies less uncertainty associated with the total investments that will be necessary to take advantage of the business opportunities that, in turn, depend on the achievements of the risk factor.



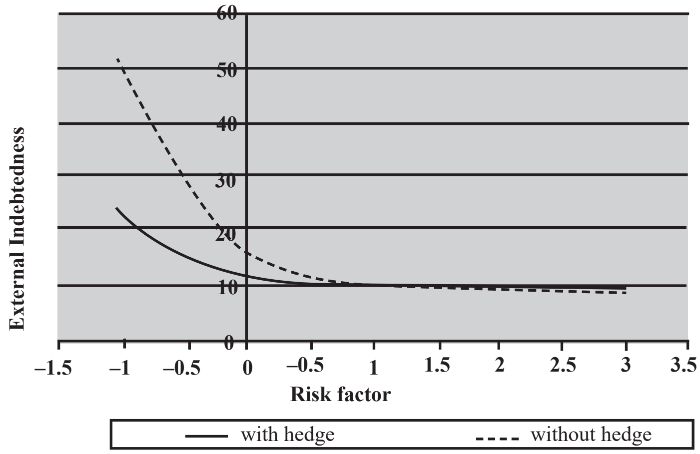
Graph 1. Optimum investment under different achievements of the risk factor.

Financing of optimal investments is done through the use of internal resources and external indebtedness. The Chart 2 shows that the company " hedged" features minor variations of internal resources, reflecting the greater stability of the optimal level of investment.



Graph 2. Internal resources under different achievements of the risk factor.

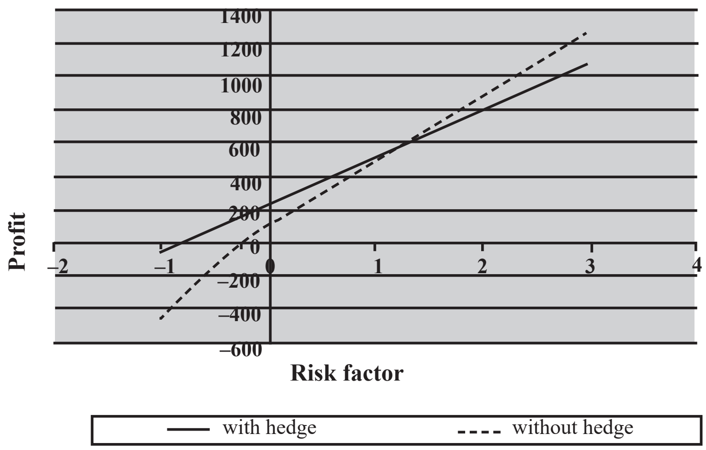
Similarly, the external debt also presents smaller variations for the company that performs the hedge.



Graph 3. External indebtedness under different achievements of the risk factor.

An important observation refers to the behaviour of the level of debt that varies with the achievements of the risk factor. Indebtedness is an integral part of both the first-order condition given by equation 19 and the company's expected profit, and therefore its impact on the optimization model is extremely relevant. In the simulation carried out, it is observed that, from a certain level of necessary investments, the company should privilege the use of resources available internally, since the debt represents a source of profit destruction. Although the hedge implies higher debt costs, for higher risk factor achievements, it also leads to a higher expected profit, since low achievements of the risk factor require, for the company not "hedged", indebtedness far superior to that of the company " hedged ".

Finally, the profits of the "hedged" and non-hedged companies can be compared through Chart 4. It can be observed that the hedge reduces the range of profit variation, smoothing the impact of the achievements of the risk factors. In terms of expected values, for the "hedged" company, the lower profit on high risk factor realizations is more than offset by the higher profit on low risk factor realizations.

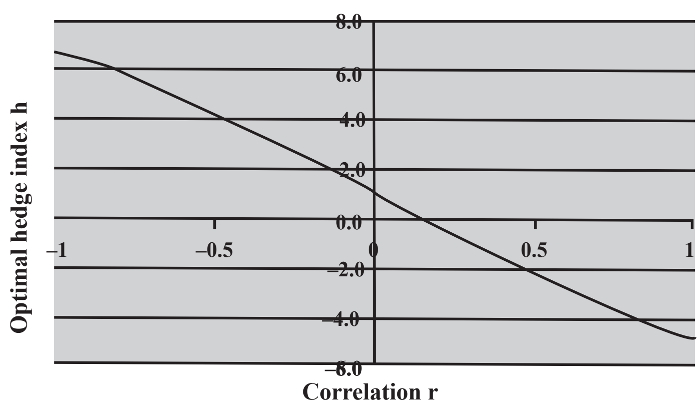


Graph 4. Profit under different achievements of the risk factor.

Thus, the numerical simulations confirm the expected results: the hedge stabilizes the investment needs and adjusts the availability of internal resources in such a way that the borrowing requirements have a lower average cost. The benefit of liquidity contributes to a greater need for hedge, but it is not a decisive factor to distinguish the functions of profit, since the inclinations of the curves of internal resources with the risk factor are similar for the hedged and non-hedged strategies.

In the study discussed, the results of the simulation were presented using a positive correlation between the risk factor and the investment opportunities. Several other simulations can also be evaluated by altering the parameters of Table 1, such as the evaluation of the optimal level of hedge as a function of the correlation of the risk factor with the investment opportunities.

It is important to note that the optimal hedge is not a linear correlation function. Although the equation 14 seems to describe a linear relationship, it should not be forgotten that the parameter θ is also a function of the correlation. The level of elimination or risk-taking that maximizes expected profit depends on the gains from investments as well as the costs of indebtedness and liquidity benefits. Thus, the behaviour of the optimal hedge is sensitive not only to the parameters of the function f and to the correlation ρ, but also to the parameters of the functions C and B and to the initial available resources w_0, as determined by equation 14. By varying the availabilities to be managed in t_0, one can find the qualitative behaviour of the optimal hedge index for different correlations, as described in Graph 5.



Graph 5. Optimum hedge ratio in function the correlation between the opportunities investment and the risk factor.

Final Comments

When considering the interaction between investment and financing decisions, aspects of indebtedness and liquidity can influence the process of value creation, since they have the potential to lead to costs or benefits for the company. If investment opportunities are dependent on a risk factor that also affects the availability of internal resources of the company, then the issue of risk management becomes relevant to the optimization of shareholder wealth. In this study, the level of hedge was identified that maximizes the hope of profit. Through an analytical evaluation, a generic formula was obtained for the optimum level of the risk management function. It is interesting to identify how investment and risk management decisions affect indebtedness and liquidity when maximizing profits is sought. To obtain the sensitivity of the analytical formula of the optimal hedge, the simulation was used. Thus, in addition to the theoretical contribution of the incorporation of several financial decisions into a single model, this research also sought to present simulations that allowed to identify relevant interrelations between the variables.

In the work, from the arbitrary definition of the functions and parameters of the model, several simulations were carried out aiming, mainly, the analysis of the sensitivity of the hedge decision due to fluctuations of other variables. Thus, analytical and complex relations between variables could be evaluated in light of the results of the simulations. The practical implementation of the described optimal hedge strategy involves the following steps: (1) identification of the company's set of investment opportunities and the relationship with the risk factor involved; (2) the identification of the cost functions arising from the indebtedness and the benefits that the company attributes to the availability of internal resources; (3) the purchase or sale of forward or futures contracts associated with the risk factor and equivalent to the calculated optimal hedge index . The reference value for optimal number of contracts is the total of available initial resources that are subject to fluctuation of the risk factor. Note, therefore, that the procedure in the derivatives market involves a simple operation with derivatives. The contribution of the model is in the refinement of the number of contracts to be traded, since the hedge strategy takes into account the correlation between the investment opportunities and the risk factor, besides the effects of additional borrowing needs and the availability of resources for investments.

It is important to highlight potential problems of practical applicability of the hedge strategy. If the hedging instrument does not have high liquidity, the transaction cost can considerably reduce the expected profits. In addition, the derivative used for the hedge may be influenced by other variables and not only the risk factor considered. For example, if the risk factor is the exchange rate, the available exchange rate derivative instruments may also be sensitive to changes in interest rates, implying a potential base risk in the hedging strategy. In addition, the instability of the correlation of the risk factor and the investment opportunities may represent an important fact to be evaluated, especially in extreme scenarios of high economic turbulence. Finally, questions of calibration of the parameters of the model to suit market data and other practical aspects such as forms of taxation can also be points of attention in the use of the model. By treating the various financial decisions in an integrated way, establishing the links between investments, financing, liquidity and risk management, the model proposed in this work has a robust characteristic and can serve as a basis for several other investigations. For example, a suggestion for future studies refers to further detailing the influence of dividend policy on risk management. The model of this paper incorporates the dividend decision in a simplistic way. All the resources available internally are used to finance the investments and, therefore, the decision of dividends entails the complete retention of the funds available. Eventually, a more realistic model could involve the issue of value creation or loss arising from the dividend strategy. In this situation,

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